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Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION OCTOBER 2011

Fifth Semester

Core Course—FUZZY MATHEMATICS

(For Model—I B.Sc. Mathematics)

Time : Three Hours Maximum Weight : 25

Part A—(Objective Type Questions)

Answer **all** *questions.*
Each bunch of **four** *questions has Weight* 1.

I. 1 Define a fuzzy set of type 2.

2 Define height of a fuzzy set.

3 Write the standard complement $\overline{A}$ of the fuzzy set $A = \cdot 4/v + \cdot 2/w + \cdot 3/x + \cdot 6/y + 1/z$.

4 Give the scalar cardinality of the fuzzy set $D(x) = 1 - \frac{x}{10}$ for $x \in \{0, 1, 2, ..., 10\} = X$.

II. 5 Define the equilibrium of a fuzzy complement C.

6 Define the dual point of a $\in [0, 1]$ with respect to a fuzzy complement C.

7 State second characterization theorem of fuzzy complements.

8 Define the pseudo inverse of an increasing generator g.

III. 9 Is every ordinary real number a fuzzy number ? Justify.

10 Calculate [4, 10]/[1, 2], where denotes the arithmetic operation of division on closed intervals.

11 If A and B are fuzzy numbers, define A*B.

12 What is the identity element 1 for the arithmetic operation of multiplication on closed intervals ?

IV. 13 Give an example of a unconditional and unqualified fuzzy proposition.

14 Write the canonical form of a conditional and unqualified proposition.

15 What are fuzzy quantifiers of the second kind ?

16 Give the schema of generalized modus tollens.

(4 × 1 = 4)

Part B—(Short Answer Questions)

Answer any **five** *questions.*
Each question has Weight 1.

17. Show that the law of contradiction is violated for fuzzy sets.

18. Give a geometrical interpretation of fuzzy subsets of a finite universal set.

Turn over

19. Prove that every fuzzy complement has atmost one equilibrium.
20. Prove that the standard intersection is the only idempotent t-norm.
21. Define a fuzzy number.
22. If A and B are closed intervals, show that $A + B = B + A$ and $A \cdot B = B \cdot A$.
23. Consider the proposition "if x is A then y is B". Let $A = .1/x_1 + .8/x_2 + 1/x_3$ and $B = .5/y_1 + 1/y_2$. Find the fuzzy set R defined by $R(x, y) = I^2[A(x), B(y)]$, where I denotes the Lukasiewiez implication.
24. Explain the compositional rule of inference from conditional fuzzy propositions.

(5 × 1 = 5)

Part C—(Short Essay Questions)

Answer any **four** *questions.*
Each question has Weight 2.

25. State and prove the first decomposition theorem of fuzzy sets.
26. For all $a, b \in [0,1]$, $i_{min}(a,b) \le i(a,b) \le \min(a,b)$ where i_{min} denotes the drastic intersection.
27. Let u_w denote the Y class of Yager t-conorms defined by $u_w(a,b) = \min\left\{1, (a^w + b^w)^{1/w}\right\}$, $w > 0$. Prove that $\max(a,b) \le u_w(a,b) \le u_{max}(a,b)$.
28. Consider two fuzzy numbers A and B defined by

$$A(x) = \begin{cases} 0, & x \le -1 \text{ and } x > 3 \\ (x+1)/2, & -1 < x \le 1 \\ (3-x)/2, & 1 < x \le 3 \end{cases}$$

$$B(x) = \begin{cases} 0, & x \le 1 \text{ and } x > 5 \\ (x-1)/2, & 1 < x \le 3 \\ (5-x)/2, & 3 < x \le 5 \end{cases}$$

Find ${}^{\alpha}(A + B)$ and ${}^{\alpha}(A - B)$.

29. Explain unconditional and qualified fuzzy propositions.
30. Let sets of values of variables x and y be $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ respectively. Assume that the proposition "if x is A then y is B" is given where $A = .5/x_1 + 1/x_2 + .6/x_3$ and $B = 1/y_1 + .4/y_2$. Given a fact expressed by the proposition "x is A' ", where $A' = .6/x_1 + .9/x_2 + .7/x_3$, use generalized ponens to derive a conclusion in the form "y is B".

(4 × 2 = 8)

Part B—(Essay Questions)

Answer any **two** *questions.*
Each question has Weight 4.

31. Given a t-norm i and an involutive fuzzy complement C, prove that the binary operation u on [0, 1] defined by $u(a, b) = c(i(c(a), c(b)))$ for all $a, b \in [0, 1]$ is a t–conorm such that $< i, u, c >$ is a dual triple.

32. State and prove a necessary and sufficient condition for $A \in \mathfrak{I}(\mathbb{R})$ to be a fuzzy number.

33. Explain fuzzy quantifiers.

$(2 \times 4 = 8)$